Experiment
27
Capacitors

The charge q on a capacitor's plate is proportional to the potential difference V across the capacitor. We express this with

$$V = \frac{q}{C},$$

where C is a proportionality constant known as the *capacitance*. C is measured in the unit of the farad, F, (1 farad = 1 coulomb/volt).

If a capacitor of capacitance C (in farads), initially charged to a potential V_0 (volts) is connected across a resistor R (in ohms), a time-dependent current will flow according to Ohm's law. This situation is shown by the RC (resistor-capacitor) circuit below when the switch is closed.

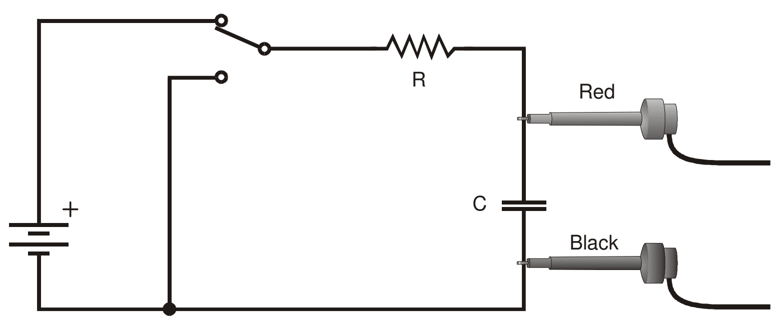

Figure 1

As the current flows, the charge q is depleted, reducing the potential across the capacitor, which in turn reduces the current. This process creates an exponentially decreasing current, modeled by

$$V(t) = V_0 e^{-\frac{t}{RC}}$$

The rate of the decrease is determined by the product RC, known as the *time constant* of the circuit. A large time constant means that the capacitor will discharge slowly.

When the capacitor is charged, the potential across it approaches the final value exponentially, modeled by

$$V(t) = V_0\left(1 - e^{-\frac{t}{RC}}\right)$$

The same time constant RC describes the rate of charging as well as the rate of discharging.

OBJECTIVES

- Measure an experimental time constant of a resistor-capacitor circuit.
- Compare the time constant to the value predicted from the component values of the resistance and capacitance.
- Measure the potential across a capacitor as a function of time as it discharges and as it charges.
- Fit an exponential function to the data. One of the fit parameters corresponds to an experimental time constant.

MATERIALS

Power Macintosh or Windows PC	10-μF non-polarized capacitor (blue found in the first long drawer my desk)
LabPro or Universal Lab Interface	100-kΩ, 47-kΩ resistors
Logger *Pro*	two C or D cells with battery holder
Vernier Voltage Probe	single-pole, double-throw switch
connecting wires	

PRELIMINARY QUESTIONS

1. Consider a candy jar, initially with 1000 candies. You walk past it once each hour. Since you don't want anyone to notice that you're taking candy, each time you take 10% of the candies remaining in the jar. Sketch a graph of the number of candies for a few hours.

2. How would the graph change if instead of removing 10% of the candies, you removed 20%? Sketch your new graph.

PROCEDURE

1. Connect the circuit as shown in Figure 1 above with the 10-μF capacitor and the 100-kΩ resistor. Record the values of your resistor and capacitor in your data table, as well as any tolerance values marked on them.

2. Connect the Voltage Probe to Channel 1 of the LabPro or Universal Lab Interface, as well as across the capacitor, with the red (positive lead) to the side of the capacitor connected to the resistor. Connect the black lead to the other side of the capacitor.

3. Open the file in the Experiment 27 folder of *Physics with Computers*. A graph will be displayed. The vertical axis of the graph has potential scaled from 0 to 4 V. The horizontal axis has time scaled from 0 to 10 s.

4. Charge the capacitor for 30 s or so with the switch in the position as illustrated in Figure 1. You can watch the voltage reading at the bottom of the screen to see if the potential is still increasing. Wait until the potential is constant.

5. Click Collect to begin data collection. As soon as graphing starts, throw the switch to its other position to discharge the capacitor. Your data should show a constant value initially, then decreasing function.

6. To compare your data to the model, select only the data after the potential has started to decrease by dragging across the graph; that is, omit the constant portion. Click the curve fit tool, and from the function selection box, choose the Natural Exponential function, A*exp(–C*x) + B. Click Try Fit, and inspect the fit. Click OK to return to the main graph window.

7. Record the value of the fit parameters in your data table. Notice that the C used in the curve fit is not the same as the C used to stand for capacitance. Compare the fit equation to the mathematical model for a capacitor discharge proposed in the introduction,

$$V(t) = V_0 e^{-\frac{t}{RC}}$$

   How is fit constant C related to the time constant of the circuit, which was defined in the introduction?

8. Print or sketch the graph of potential *vs*. time. Choose Store Latest Run from the Data menu to store your data. You will need this data for later analysis.

9. The capacitor is now discharged. To monitor the charging process, click Collect. As soon as data collection begins, throw the switch the other way. Allow the data collection to run to completion.

10. This time you will compare your data to the mathematical model for a capacitor charging,

$$V(t) = V_0\left[1 - e^{-\frac{t}{RC}}\right]$$

Select the data beginning *after* the potential has started to increase by dragging across the graph. Click the curve fit tool, , and from the function selection box, choose the Inverse Exponential function, A*(1 – exp(–C*x)) + B. Click Try Fit and inspect the fit. Click OK to return to the main graph window.

11. Record the value of the fit parameters in your data table. Compare the fit equation to the mathematical model for a charging capacitor.

12. Hide your first runs by choosing Hide Run ▸ Run 1 from the Data menu. Remove any remaining fit information by clicking the gray close box in the floating boxes.

13. Now you will repeat the experiment with a resistor of lower value. How do you think this change will affect the way the capacitor discharges? Rebuild your circuit using the 47-kΩ resistor and repeat Steps 4 – 11.

DATA TABLE

	Fit parameters				Resistor	Capacitor	Time constant
Trial	A	B	C	1/C	R (Ω)	C (F)	RC (s)
Discharge 1							
Charge 1							
Discharge 2							
Charge 2							

ANALYSIS

1. In the data table, calculate the time constant of the circuit used; that is, the product of resistance in ohms and capacitance in farads. (Note that 1ΩF = 1 s).

2. Calculate and enter in the data table the inverse of the fit constant *C* for each trial. Now compare each of these values to the time constant of your circuit.

3. Note that resistors and capacitors are not marked with their exact values, but only approximate values with a tolerance. Ask your instructor the tolerance of the resistors and capacitors you are using. If there is a discrepancy between the two quantities compared in Question 2, can the tolerance values explain the difference?

4. What was the effect of reducing the resistance of the resistor on the way the capacitor discharged?

5. How would the graphs of your discharge graph look if you plotted the natural logarithm of the potential across the capacitor *vs*. time? Sketch a prediction. Show Run 1 (the first discharge of the capacitor) and hide the remaining runs. Click on the *y*-axis label and select ln(*V*). Uncheck the boxes for the Potential column. Click OK to see the new plot.

6. What is the significance of the slope of the plot of ln(*V*) *vs.* time for a capacitor discharge circuit?

EXTENSIONS

1. What percentage of the initial potential remains after one time constant has passed? After two time constants? Three?

2. Use a Vernier Current & Voltage Probe System to simultaneously measure the current through the resistor and the potential across the capacitor. How will they be related?

3. Instead of a resistor, use a small flashlight bulb. To light the bulb for a perceptible time, use a large capacitor (approximately 1 F). Collect data. Explain the shape of the graph.

4. Try different value resistors and capacitors and see how the capacitor discharge curves change.

5. Try two 10-µF capacitors in parallel. Predict what will happen to the time constant. Repeat the discharge measurement and determine the time constant of the new circuit using a curve fit.

6. Try two 10-µF capacitors in series. Predict what will happen to the time constant. Repeat the discharge measurement and determine the time constant for the new circuit using a curve fit.